647510 **MSC1MatE101x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Jan/Feb.-2022 [NEW COURSE]
Classical Mechanics 1(Elective (New))

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q-1 (A) Answer any Two out of Three **[10]**

(1) Using D'Alemberts principle derive Lagrange's equations of motion.

(2) State types of constraints. Define Rheonomic constraint and give an appropriate example.

(3) Show that Lagrange's equations of motion can also be written as

$$\frac{\partial \dot{T}}{\partial \dot{q}_j} - 2\frac{\partial T}{\partial q_j} = Q_j$$

Q-1 (B) Answer any One out of Two **[4]**

(1) Obtain Lagrange's equations of motion for simple pendulum.

(2) In usual notations prove that $T = T_0 + T_1 + T_2$.

Q-2 (A) Answer any Two out of Three **[10]**

(1) Derive the condition for extremum of the line integral $\int_{x_1}^{x_2} f(y, \dot{y}, x)dx$.

(2) Show that the catenary curve $x = a\cosh\frac{y-b}{a}$ gives the minimum surface area of revolution about y-axis.

(3) Discuss law of conservation of angular momentum of a system using Lagrangian formalism.

Q-2 (B) Answer any One out of Two **[4]**

(1) State Hamilton's principle and derive Lagrange's equations from it.

(2) Obtain energy function for Lagrangian given by

$$L = \frac{m}{2}l^2\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) + mgl\cos\theta$$

Q-3 (A) Answer any Two out of Three **[10]**

(1) Obtain Lagrangian for a central force two body problem.

(2) Stating Lagrangian for a central force two body problem obtain first integrals.

(3) Using Lagrangian formalism obtain differential equation of orbit for a central force two body problem.

Q-3 (B) Answer any One out of Two **[4]**

(1) Find the force law for the circular orbit $r = \cos\theta$.

(2) State differential equation of orbit. For inverse square law of force, obtain condition for elliptic orbit.

Q-4 (A) Answer any Two out of Three **[10]**

(1) Show that the general displacement of a rigid body with one point fixed is a rotation about some axis passing through the fixed point.

(2) Obtain the Euler's equations of motion of a rigid body with one point fixed.

(3) Show that the components of angular velocity vector along the body set of axes are given by

$$\omega_{x'} = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi$$
$$\omega_{y'} = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi$$
$$\omega_{z'} = \dot{\phi}\cos\theta + \dot{\psi}$$

Q-4 (B) Answer any One out of Two **[4]**

(1) Discuss constraints in a rigid body. Hence obtain degrees of freedom.

(2) Show that infinitesimal rotation of a rigid body with one point fixed is commutative.

Q-5 (A) Answer any Two out of Three **[10]**

(1) Giving all details, obtain Lagrange's equations of motion for Atwood's machine.

(2) For the following Lagrangian, obtain all generalized memoneta. Which of them are conserved? Why?

$$L = \frac{I_1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2 - mgl\cos\theta$$

(3) Derive $T = \frac{1}{2}I\omega^2$ notations being usual.

Q-5 (B) Answer any One out of Two **[4]**

(1) Show that if F does not depend on x explicitly, $y \equiv y(x)$ and $y' = \frac{dy}{dx}$ then condition for the extremum of the line integral $\int F(y, y')dx$ can be given by $F - y'\frac{\partial F}{\partial y'} = constant.$

(2) State Kepler's laws for planetary motion.

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